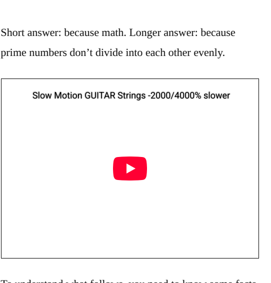
Why can't you tune your guitar?

Ethan · May 27, 2019 · 16 Comments

Short answer: because math. Longer answer: because prime numbers don't divide into each other evenly.



To understand what follows, you need to know some facts about the physics of vibrating strings:

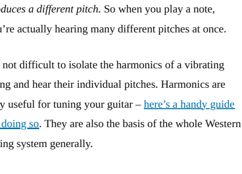
- When you pluck a guitar string, it vibrates to and fro. You can tell how fast the string is vibrating by listening to the pitch it produces.
- Shorter and higher-tension strings vibrate faster and make higher pitches. Longer and lower-tension strings vibrate slower and make lower pitches.
- The scientific term for the rate of the string's vibration is its *frequency.* You measure frequency in *hertz (Hz)*, a unit that just means "vibrations per second." The standard tuning pitch, 440 Hz, is the pitch you hear when an object (like a tuning fork or guitar string) vibrates to and fro 440 times per second.
- Strings can vibrate in many different ways at once. In addition to the entire length of the string bending back and forth, the string can also vibrate in halves, in thirds, in quarters, and so on. These vibrations of string subsections are called *harmonics* (or *overtones*, or *partials*, they all mean the same thing.)

If you watch slow-motion video of a guitar string vibrating, you'll see a complex, evolving blend of squiggles. These squiggles are the mathematical sum of all of the string's different harmonics. The weird and interesting thing about harmonics is that *each one produces a different pitch.* So when you play a note, you're actually hearing many different pitches at once.

It's not difficult to isolate the harmonics of a vibrating string and hear their individual pitches. Harmonics are very useful for tuning your guitar – here's a handy guide for doing so. They are also the basis of the whole Western tuning system generally.

The math of harmonics is really simple

As a string vibrates, its longer subsections produce lower and louder harmonics, while its shorter subsections produce higher and quieter harmonics. Click the image below to hear the first six harmonics of a string:



Remember that in a real-world string, you are hearing all these harmonics blended together. However, you can isolate the harmonics of a guitar string by lightly touching it in certain places to deaden some of the vibrations.

- If you touch the vibrating string at its halfway point, that deadens the vibration along the string's entire length, enabling you to hear it vibrating in halves.
- If you touch the string a third of the way along its length, that deadens the vibration both of the entire string and the halves of the string, so you can now hear it vibrating in thirds.
- If you touch the string a quarter of the way along its length, that deadens the vibration of the whole string, the halves, and the thirds, so you can now hear it vibrating in quarters.

Harmonics give you a good-sounding collection of pitches

Imagine that you have a guitar string tuned to play a note called "middle C," which has a frequency of 1 Hz. (In reality, middle C has a frequency of 261.626 Hz, so if you want to think in terms of actual frequencies, just multiply all the numbers in the following paragraphs by 261.626.)

The **first harmonic** is **the string vibrating along its entire length,** otherwise known as the *fundamental frequency.* When we say that your C string is vibrating at 1 Hz, that really means that its *fundamental* has a frequency of 1 Hz. The other harmonics all have other frequencies, and we'll get to those, but the fundamental is usually the loudest harmonic, and it's usually the only one you're aware of hearing.

The **second harmonic** is the one you get from **the string vibrating in halves.** Each half of the string vibrates at twice the frequency of the whole string. The 2:1 relationship between the pitches of the first and second harmonics is called an *octave*. (I know that the word suggests the number eight, not the number two. Don't worry about it.) The pitch that's an octave above middle C has a frequency of 2 Hz, and it is also called C. Both of these notes have the same letter name because in Western convention, notes an octave apart from each other are considered to be "the same note". The important concept here is that you can move up an octave from any pitch by doubling its frequency. You can also move down an octave from any pitch by halving its frequency.

The **third harmonic** is the one you get from **the string vibrating in thirds.** Its frequency is three times the fundamental frequency. Since your C string's fundamental is 1 Hz, the third harmonic has a frequency of 3 Hz, and it produces a note called G. The interval between C and G is called a perfect fifth, for reasons having nothing to do with harmonics. I know it's confusing.

The **fourth harmonic** is the one you get from **the string vibrating in quarters,** at 4 Hz. This note is an octave higher than the second harmonic, and so is also called C. (The eighth harmonic will also play C, as will the sixteenth, and the thirty-second, and all the powers of two up to infinity.)

The **fifth harmonic** is the one you get from **the string vibrating in fifths.** Its frequency is 5 Hz, and it produces a note called E. The interval between C and E is called a major third, which is another name that has nothing to do with harmonics.

There are many more harmonics (infinitely many more, in theory) but these first five are the most audible ones.

E (5 Hz)
C (4 Hz)
G (3 Hz)
C (2 Hz)
C (1 Hz)

The ancient Greeks figured out that if you have a set of strings, it sounds really good if you tune them following the pitch ratios from the natural harmonic series. In such tuning systems, you pick a starting frequency, and then multiply or divide it by ratios of whole numbers to generate more frequencies, the same way you figure out the frequencies of a single string's harmonics. The best-sounding note combinations (to Western people) are the ones derived from the first few harmonics. In other words, you get the nicest harmony (for Western people) when you multiply and divide your frequencies by ratios of the smallest prime numbers: 2, 3, and 5.

So, let's do it. Let's make a tuning system based on the harmonics of your C string. First, we should find the C, G and E notes whose frequencies are as close to each other as possible.

- We've already got C at 1 Hz.
- We can bring our G at 3 Hz down an octave by dividing its frequency in half. This gives us a G at 3/2 Hz.
- We can also bring our E at 5 Hz down two octaves by dividing its frequency in half twice. This gives us an E at 5/4 Hz.

When you play 1 Hz, 5/4 Hz and 3/2 Hz at the same time, you get a lovely sound called a C major triad.

So far, so good. Let's find some more notes!

We can extend our tuning system by thinking of G as our base note, and looking at *its* harmonics. When we do, we get two new notes. The third harmonic of G is D at 9 Hz. (Thanks to octave equivalency, we can also make Ds at 9/2 Hz, and 9/4 Hz, and 9/8 Hz, and 18 Hz, and 36 Hz, and so on.) The fifth harmonic of G is B at 15 Hz. (There are also Bs at 15/2 Hz, and 30 Hz, and so on.)

The notes C and G feel closely related to each other because of their shared harmonic relationship. The chords you get from their respective overtone series also feel related. If you alternate between C major and G major chords, it just about always sounds good.

Now let's extend our tuning system further by treating D as our base note. The harmonics of D give us two more new notes: the third harmonic is A at 27 Hz (and 27/2 Hz and 27/4 Hz and 27/8 Hz), and the fifth harmonic is F-sharp at 45 Hz (and 45/2 Hz and 45/4 Hz and 45/8 Hz).

G major chords and D major chords have the same relationship as C major and G major chords, and they sound equally good when you alternate them. Also, C major, G major and D major chords all sound good as a group, in any order and any combination. Western people just really like the sound of shared harmonics. Last thing: notice that you can combine the harmonics of C, G and D to form a G major scale.

Now let's make some more notes by treating A as our base and looking at *its* harmonics. The third harmonic of A is E at 81 Hz (and 81/2 Hz and 81/4 Hz etc).

But wait. We already had an E, at 5 Hz. If we put these two E's in the same octave, then one of them is at 80/64 Hz, and the other is at 81/64 Hz. That may not seem like much of a difference, but even untrained listeners will be able to hear that they are out of tune with each other. Furthermore, if we use the E derived from C, then it will be out of tune with A. However, if we use the E derived from A, then it will be out of tune with C. This is going to be a problem.

Let's forget about that conflict for a second. Instead, we'll try a different method of expanding our tuning system, by going in the opposite direction from C. Let's think about a note that contains C in *its* harmonic series. That would be F at 1/3 Hz. The third harmonic of F is C at 1 Hz, as expected. The fifth harmonic of F is A at 5/3 Hz.

Uh oh. This new A conflicts with the one we already had at 27 Hz. That is not good. But let's bracket that and keep expanding.

We can push further left by finding the note whose overtone series contains F. That would be B-flat at 1/9 Hz. Its third harmonic is F at 1/3 Hz, and its fifth harmonic is D at 5/9 Hz. And now we have a new problem: this D clashes with our existing D at 9 Hz.

Can you see the pattern here? Anytime you want to use intervals based on third harmonics, you're multiplying and dividing by 3, but anytime you want to use intervals based on fifth harmonics, you're multiplying and dividing by 5. (Notice that the conflicting notes always conflict by the same amount, too, a ratio of 81/80.) Starting from C, it's possible to produce any note if you multiply or divide your frequencies by 3 enough times, but those notes won't be in tune with the notes you'd get multiplying or dividing your frequencies by 5, because 3 and 5 don't mutually divide evenly. This is not just an abstract mathematical issue. It's the reason that it's impossible to have a guitar be in tune with itself.

Guitars can't be perfectly in tune

Imagine that the guitar's low E string has a frequency of 1 Hz. (It's really 82.4069 Hz; feel free to multiply everything in this next section by that number if you want actual frequencies.) Ideally, you want your high E string to be tuned two octaves above the low one, at 4 Hz. Let's see if you can get there by tuning the strings pairwise.

- The interval between E and A is a fifth, but it's upside down, because we're going *down* a fifth from E. In music theory terms, an upside down fifth is called a fourth. You go up a fourth by multiplying your frequency by 4/3 (it's 3/2 upside down, doubled to bring it up an octave.) So your A string is now tuned to 4/3 Hz.
- The D string should be another fourth higher, so you can multiply by 4/3 again, giving you 16/9 Hz.
- The G string should be yet another fourth higher, so you multiply by 4/3 to get 64/27 Hz.
- The B string is a major third higher than G, which means multiplying by 5/4, and that puts you at 80/27 Hz.
- Finally, to get to high E, that's another fourth, so you multiply by 4/3 again, giving you… uh… 320/81 Hz.

This is not good. We wanted the high E to be at 4 Hz, which is the same as 324/81 Hz. We're 4/81 Hz flat! That difference is big enough to make your guitar tuning sound like warm garbage.

Let's try a different strategy. I said you should tune the B string a major third above G. However, you could just as easily retune the B string so it's a fifth plus an octave above the low E string. You do this by multiplying 1 Hz by 3/2, and then doubling it, which puts your B at 3 Hz. Now the B string sounds perfectly in tune with the low E string at 1 Hz, and with the high E string at 4 Hz. Unfortunately, the B string is now out of tune with the G string at 64/27 Hz.

So maybe you should just retune the G string a major third below your new B, at 12/5 Hz. That makes the G and B strings sound great together. Unfortunately, now the G string is out of tune with the D string at 16/9 Hz.

You could retune the D string to be a fourth below G… but now the D string will be out of tune with the A string. If you retune the A string based on your new D, then it will be out of tune against the low E string. And if you retune the low E string based on your new A, then it will be out of tune with the high E string.

The bottom line: *there is no way to tune the guitar so that every string is in tune with every other string.*

The standard tuning system is an unsatisfying compromise

The mathematical awkwardness of harmonics-based tuning systems has caused Western musicians a lot of pain over the past thousand years. Depending on your starting pitch, some intervals can be perfectly in tune, but others can't be. And the more harmonically complex you want your music to be, the worse the tuning issues become.

In the 16th century, Chinese and Dutch musicians independently came up with an alternative system to harmonics-based tuning, called 12-tone equal temperament, or 12-TET. It's the system that the entire Western world uses today. The idea behind 12-TET is to have everything be pretty much in tune, which you accomplish by having everything be a little bit *out* of tune. Is this a worthwhile compromise? Let's do the math and find out.

In 12-TET, you divide up the octave into twelve equally-sized semitones (the interval between two adjacent piano keys or guitar frets). To go up a semitone from any note, you multiply its frequency by the 12th root of 2 (about 1.05946). To go down a semitone from any note, you divide its frequency by the 12th root of 2. If you go up by an octave (twelve semitones), you're multiplying your frequency by the 12th root of 2 twelve times, which works out to 2. That's a perfect octave, hooray! Unfortunately, you can't exactly create the other harmonics-based intervals by adding up 12-TET semitones; you can only approximate them.

Remember that the pure fifth you get from harmonics is a frequency ratio of 3/2. In 12-TET, however, you make a fifth by adding up seven semitones. This means that you multiply your frequency by the 12th root of two seven times, which comes to about 1.498. That's close to 3/2, but it's not exact. As a result, fifths in 12-TET sound a little flat compared to what your ear is expecting from natural harmonics.

Major thirds are worse in 12-TET. Recall that the major third you get from the overtone series is a frequency ratio of 5/4. In 12-TET, you make a major third by adding four semitones, which means that you multiply your frequency by the 12th root of 2 four times. That comes to 1.25992, which is noticeably higher than 5/4. Thirds in 12-TET are quite sharp compared to what your ears are expecting from natural harmonics.

If thirds and fifths are so out of tune in 12-TET, why do we use it? The advantage is that all the thirds and fifths in all the keys are out of tune *by the same amount.* None of them sound perfect, but none of them sound terrible, either. You don't have to worry about whether your notes are derived from the third harmonic of some note or the fifth harmonic of some other note; they all just work together, kind of. If you use a digital guitar tuner, you are tuning your strings to the 12-TET versions of E, A, D, G and B. None of them will be perfectly in tune with each other, but they will all be wrong by an acceptable amount. Also, songs in the key of E won't sound any better or worse than songs in the key of F or E-flat.

Not everyone in history thought that 12-TET was an acceptable compromise. Johann Sebastian Bach thought we should use other tuning systems that made better-sounding thirds and fifths in some keys in exchange for worse-sounding thirds and fifths in others. In Bach's preferred tuning, each key had its own distinctive blend of smoothness and harshness. However, Bach did not get his way. We as a civilization have collectively decided that we want all our keys to be interchangeable. There are good reasons to want this! In 12-TET, all intervals and chords are built from standardized, Lego-like parts. You don't have to keep track of a complicated web of different-sized intervals in every key. If you move a song from C to C-sharp or D or anywhere else, you can be confident that it will still sound "the same."

You don't have to use 12-TET, though

Some musicians don't want to accommodate to 12-TET, insisting instead that we should continue to use pure intervals derived from harmonics the way God and Pythagoras intended. Harmonics-based tuning systems are collectively known as just intonation systems. This is a poetically apt term, because it implies fairness. By contrast, the implicit message of 12-TET is that life isn't fair. Just intonation systems give you some lovely pure intervals, but you can't change keys unless you retune all your instruments. In other world cultures, this is not necessarily a problem. Hindustani classical music uses just intonation over an omnipresent drone, so everything is always in the same "key."

Meanwhile, a few Western oddballs and nerds have explored just intonation systems that use bigger prime numbers than 2, 3 and 5 to generate finer pure intervals. Harry Partch used the primes up to eleven to make a tuning system that divides up the octave into 43 pure parts rather than 12 impure ones. You can try the Partch 43-tone scale using the Wilsonic app or Audiokit Synth One. It's extremely strange! But, I guess, it's strange in a pure way. I have made some music of my own with exotic just intonation tunings.

Just intonation may also play a role in the blues. There is a theory that the blues originates from the natural overtone series of I and IV. If this is true, then the characteristic chords and scales of the blues are really 12-TET approximations of the original just intonation blues scale. It's conventional to say that blues musicians and singers bend notes to make them go out of tune, but it may be that they are actually bending the 12-TET pitches to get them *in tune* instead.

Anyway, outside of the blues and the avant-garde, most Western musicians just live with everything being a little out of tune. If you're a guitarist, you know that no matter how you tune your guitar, it won't stay in tune for long anyway, so how much does any of this even matter? There's a joke among guitarists: we spend half our lives tuning, and the other half wishing we were in tune. There are lots of reasons why tuning is hard: you might be hampered by having a poorly made guitar, or by having a guitar that's not set up correctly, or by using old worn-out strings, or by changes in temperature or humidity, or just by a lack of patience or time. At least you can be secure in the knowledge that some of your tuning struggles are due to the basic unfairness of the universe, and not just the limitations of your ears or your equipment.

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16 Comments

Doug DesCombaz
March 8, 2026 at 2:59 pm

True. Thank you. I want to add additional nuance regarding a common conflation of Western music with 12-TET.

Western music is oft conflated with 12-TET, and non-recognition of microtonal playing (besides blues). 12-TET is a dominant convenience, but I don't think it accurately describe the breadth of Western music. Nor, a sometimes implied opposite that other cultures' music isn't based upon the same natural harmonics of waves.

The physics of harmonics is universal; it extends beyond human culture to all mammals, other species, and the fundamental physics of the stars. Non-Western scales are derived from these same natural harmonics. While the appreciation of dissonance and consonance is culturally conditioned through repetition and exposure (ie, emotions can be evoked by waves, but it is how those emotions are correlated with positive/negative that makes them culture), the biological baseline remains: dissonance signals tension (the "clank" that demands attention), while consonance provides resolution.

The "out-of-tune" math of 12-TET primarily plagues fretted or keyed instruments. Musicians on "open" instruments are not similarly locked in:
– Horns: Brass and woodwinds are physically built upon the natural harmonic series; 12-TET is actually unnatural for them.
– Guitars: Even on a fretted instrument, a guitarist can tweak tuning between songs to favor the "pure" intervals of a specific key.

What is often called "feel" is usually the musician's ear correcting (or intentionally un-correcting even further) the mathematical compromise of 12-TET. It's definitely not limited to the blues or guitars. When a guitarist squeezes a string –ever so gently that it doesn't even count as a bend or something the guitarist even knows they are doing –or a horn player adjusts their blow, they are playing with how that pitch harmonizes. There is truth in the simplification of Western tuning, but in practice, it is far more nuanced.

Reply

Ethan
March 8, 2026 at 3:11 pm

Ever since I started looking into this, I have had music theorists telling me that the whole just intonation thing is overblown and that 12-TET is fine, while also having singers, string players and horn players tell me, oh yeah, we are constantly fixing our thirds on the fly.

Reply

Doug DesCombaz
March 8, 2026 at 3:24 pm

Hey! Interesting. Thanks for the ref.

I'm not a lawyer, but I'm pretty sure that a court would probably find a court's order to be a "law" for purposes of the Supremacy Clause, and so you can't "ignore" it because it's unconstitutional.

Reply

Allison
August 2, 2013 at 10:14 pm

You know it's not as clear as that because the judges in every state have sworn an oath to uphold the state constitution. Thus, if a judge thinks a law violates the state constitution, he is sworn to ignore it. It's not as cut and dry as you make it out to be.

Reply

jackal
August 2, 2013 at 10:14 pm

Happy birthday, Sam! Sorry for the birthday spoiler, but hey, it sucks to be you on your birthday. Anyway, I'm sure you'll get over it. Happy birthday!

PS: [illegible] http://www.[illegible]

Reply

Allison
August 2, 2013 at 10:14 pm

Don't worry, Sam. It's okay. Lots of people forget your birthday. Just wait for them to remind them of it. Or, if you don't want to wait, you can remind them yourself. Or, if you don't want to remind them yourself, you can just wait for them to remember it on their own. Or, if you don't want to wait for them to remember it on their own